Fundamentals of a Vertical Free Jet

Moshe Alamaro
alamaro@alum.mit.edu

Introduction

The following is an outline for a new system for updrafts formation that could be used for the disruption of atmospheric inversions and a variety of applications, including smog and fog dispersion and weather modification. This analysis could lead to a few design alternatives.

The new concept will require turbomachinery that could consist of activated and retrofitted inexpensive retired de-commissioned jet engines.

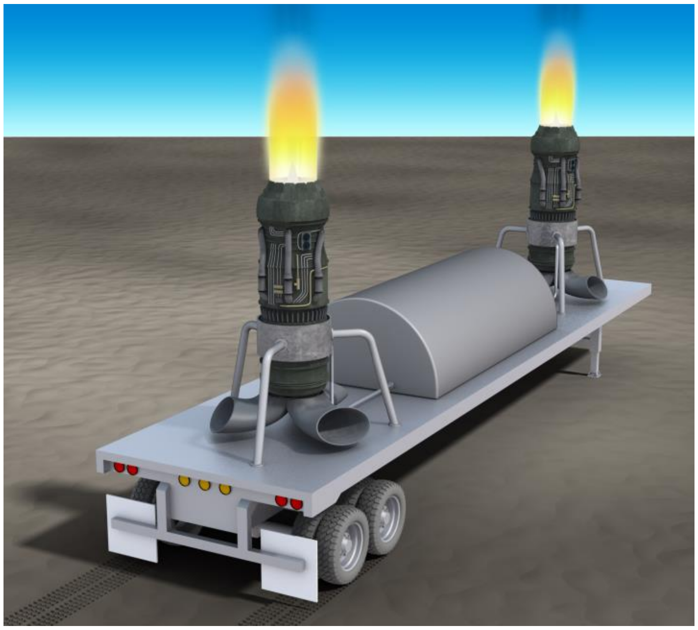

Figure 1: Artistic view of mobile jet engines mounted vertically

In my proposal, I will assume that all the turbomachinery steps are isentropic with efficiency of 100%. In the future, if and when this program is activated in China and elsewhere we will employ turbomachinery practice engineers to provide a rigorous analysis and specific design.

Definition

The following contains basic information, assumptions and calculations for a free jet produced by a turbofan jet engine for a variety of applications.

A “Free Jet” is a flow of one fluid into another. The other is surrounding fluid at rest or in a motion, relative to the jet. Walls or ducts do not confine the free jet. The jet flow is impeded only by shear stress with the surrounding fluid.

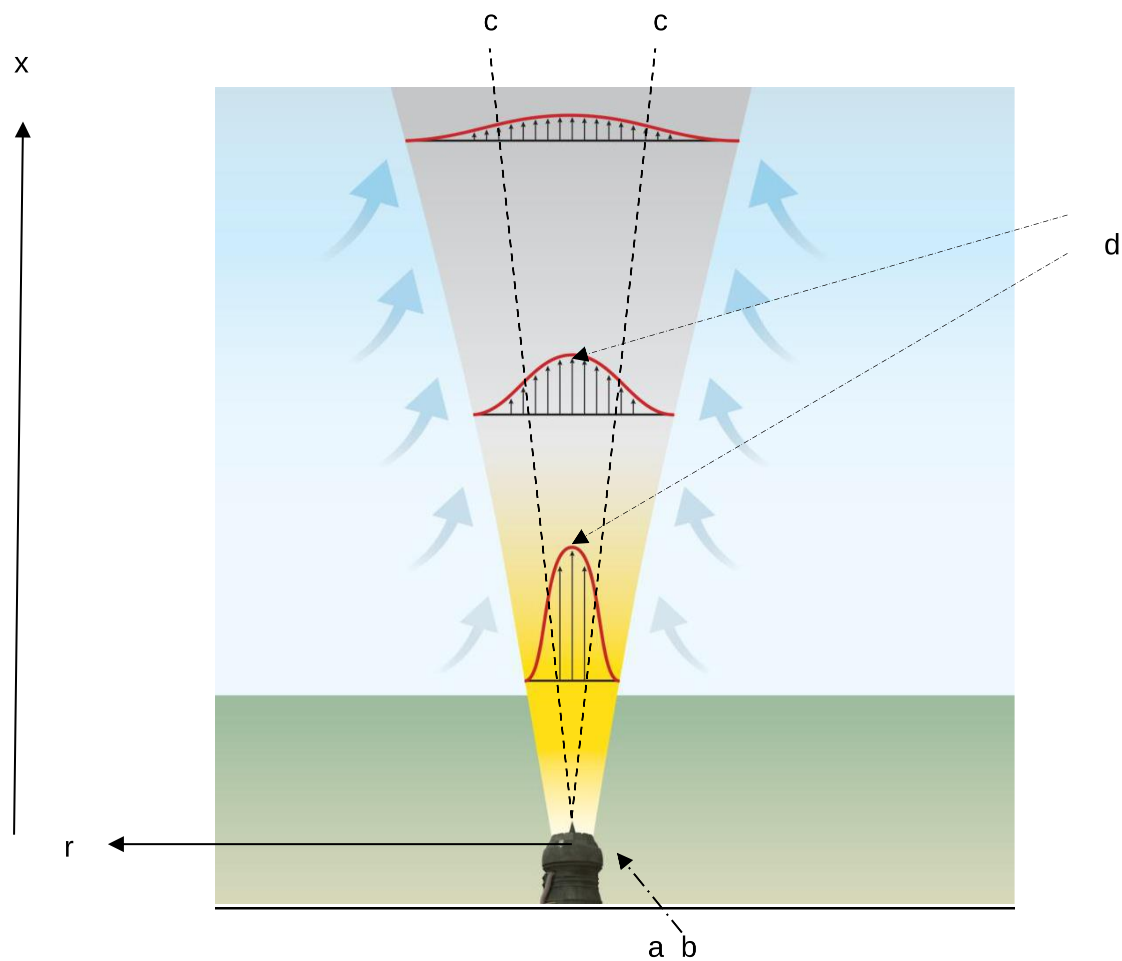

Figure 2: Geometry of a circular jet

The term “compressible” refer to the velocity of the free jet. For example, the jet can be made up of air, which is a compressible gas. However, the flow can be compressible or incompressible. In any flow where the Mach Number (M) is less than 0.3 the flow is incompressible. At $M > 0.3$ the flow is compressible. For air at ambient temperature, the flow is incompressible when the air speed is lower than approximately $100\ ms^{-1}$. The flow from a turbojet engines used in aircrafts is usually at around sonic speed, so it is initially compressible. Turbulent mixing and entrainment of air that joins the jet reduces the velocity of the compressible jet generated by a jet engine. The free jet, then, becomes incompressible quite immediately after being ejected from the nozzle.

The following analysis is intended to provide reasonable assumptions for a vertical free jet using a Pratt and Whitney JT9D turbofan. The engine can be placed horizontally and a duct could direct the jet upward. The methodology provided herein will be applied in the future to any specific propulsion systems and any specific design.

Geometry of Circular Jet

The schematic in Figure 2 describes the following:

a. Jet engine and a nozzle of diameter d_0 and cross sectional area A.

b. Approximately uniform velocity profile or “plug flow” at the jet nozzle exit.

c. Divergence lines where the flow velocity is half of the maximum velocity in the Gaussian profile or the lines of $U_{m\,0.5}$.

d. The maximum velocity U_m at the center of the circular Gaussian velocity profile.

For the vertical circular jet, the cylindrical coordinates are x and r where x is the height from the center of the nozzle. Assuming that the jet engine operates at a sea level, x is also the elevation above sea level.

Fundamental Assumptions

Buoyancy jets that extend to high altitudes, such as a few kilometers, have been studied extensively. Examples are the plums from the smokestack of a power plant. A horizontal convection jet produced by a jet engine has also been studied for understanding, for example, the turbulent wakes created by airplanes’ jet engines during takeoff and landing. Vertical convection jets that extend to a height of a few kilometers have not been studied extensively. All or most past investigations pertain to jets that penetrate a constant density of fluid. In our case, the density of air is dependent on the height or distance from the jet origin.

Extensive analysis of a turbulent jet and its geometry is given in references 1, 2, and 3. For a circular jet, the radius at the points where the air velocity in the Gaussian profile is half of the maximum velocity is given by:

$$r_{U_{m\,0.5}} \cong 0.086 \cdot x \qquad (1)$$

The thrust produced by the jet engine is the momentum flow (also called momentum flux by a few authors) produced by the jet (ignoring the thrust that is generated by pressure imbalances). The momentum flow at the exit from the nozzle is:

$$J = \dot{m}U_n = \rho_n\, A_n\, U_n^2 \qquad (2)$$

Where U_n is the speed at the nozzle exit from the engine which is approximated as uniform, A_n is the cross sectional area of the nozzle and ρ_n is the density of the gas at the nozzle exit. Here we ignore the portion of the thrust due to the fact that the static pressure of the flow is not necessarily equal to the ambient pressure.

The jet ejected from the nozzle forms a Gaussian velocity profile at certain distance from the nozzle which is estimated as $\approx (6-10)\, d_0$ while d_0 is the nozzle diameter. The jet ejected from an aviation jet engine nozzle could be supersonic, subsonic or sonic. The static pressure and the temperate of the gas ejected from the nozzle could be greater, lower or equal to the pressure of the ambient. All of these considerations are necessary for an aviation jet engine. However, our systems are stationary. At distance of $x \approx 10 \cdot d_0$ from the nozzle and by the time the flow becomes Gaussian, we assume (and show later) that the pressure and the temperature of the flow and its entrained air become equal to that of the ambient air due to entrainment and turbulent mixing.

Abramovich (ref. 1) has shown that for a circular axisymmetric jet:

$$\frac{\partial}{\partial x} J(x) = 2\pi\rho \frac{\partial}{\partial x} \int_0^\infty rU^2\,(x,r)dr = 0 \qquad (3)$$

This and a control volume analysis shows that the momentum flow of the jet remains constant at any distance x (or height) from the engine nozzle.

The thrust generated by a jet engine at cruising airplane flight is comprised of the positive momentum flow of the combustion gas emitted from the engine's nozzle, minus the negative momentum flow (ram drug) of the air mass that enters the engine at the speed of the flight. In our case, however where the engine is stationary, the only momentum flow (equal to the thrust) is generated by the flow emitted from the nozzle. In this respect, the thrust generated by our jet is similar to the thrust during takeoff, which is much larger than the thrust of jet engine in a cruising flight.

Using a control volume analysis on the jet, it is possible to show that at any arbitrary distance x from the jet origin the momentum flow of the jet is preserved. Therefore:

$$J = 2\pi\rho(x) \int_0^\infty rU^2\,(x,r)dr = constant \qquad (4)$$

for any value of x . In fact, this is the central feature of a free jet.

In Eq. (3) and (4) J is the momentum flow. In our turbofan used for analysis we assume that the density at the nozzle is equal to the density of dry air at standard sea level, where $T \cong 288K$ so $\rho_n = \rho_0 \cong 1{,}225\ kg/m\text{^}3$. $\rho(x)$ is the density at height x . $U(x,r)$ is the gas velocity which has a circular Gaussian velocity profile.

Description of a Circular Jet

The flow exiting from the jet engine nozzle has almost uniform velocity profile and therefore it is called "plug flow". Non-dimensional analysis provides the equations that approximate the subsequent Gaussian profile flow as:

$$U_m(x) = c_2\, \frac{\sqrt{J/_{\rho(x)}}}{x} \qquad (5)$$

Where $U_m(x)$ is the jet maximum velocity at the centerline, J is the momentum flow defined in Eq. (4), $\rho(x)$ is the air density as a function of height x and c_2 is some constant.

The velocity distribution as a function of r has several forms in different references. One convenient Gaussian velocity profile is:

$$U(x,r) = U_m(x) \cdot exp\left(\frac{-r^2}{a^2(x)}\right) \qquad For\ a\ circular\ jet \qquad a(x) = c_1 \cdot x \qquad (6)$$

c_1 is some constant.

To find c_1 and c_2 integrate the momentum flow expression:

$$J = 2\pi\rho(x)\int_0^\infty rU^2\,(x,r)dr = 2\pi\rho(x)\int_0^\infty r \cdot \left[c_2\,\frac{\sqrt{J/_{\rho(x)}}}{x} \cdot\ exp\left(\frac{-r^2}{a^2(x)}\right)\right]^2 \cdot dr$$

$$J = 2\pi\rho(x)c_2^2\,\frac{J}{\rho(x)x^2}\int_0^\infty r \cdot exp\left(\frac{-2r^2}{c_1^2x^2}\right)dr = 2\pi\rho(x)c_2^2\,\frac{J}{\rho(x)x^2} \cdot \frac{c_1^2x^2}{4} \;\rightarrow$$

$$\rightarrow \quad c_1^2c_2^2 = \frac{2}{\pi} \rightarrow c_2 = \frac{1}{c_1}\sqrt{2/_{\pi}} \qquad (7)$$

c_1 has been found empirically to be 0.103 so $c_2 = 7.75$ (Ref. 3).

Therefor for a circular Jet:

$$U(x,r) = 7.75\,\frac{\sqrt{J/\rho(x)}}{x}\cdot exp\left(\frac{-r^2}{a^2(x)}\right) = 7.75\,\frac{\sqrt{J/\rho(x)}}{x}\cdot exp\left(\frac{-r^2}{0.103^2x^2}\right) \quad (8)$$

The total mass rate of the jet as a function of x is:

$$\dot{m}(x) = 2\pi\rho(x)\int_0^\infty r\,U(x,r)dr = 2\pi\cdot\,7.75\,\rho(x)\frac{\sqrt{J/\rho(x)}}{x}\int_0^\infty r\cdot\,exp\left(\frac{-r^2}{c_1^2x^2}\right)dr \quad (9)$$

Using:

$$\int_0^\infty r\cdot exp\left[\frac{-r^2}{a}\right]dr = \frac{a}{2} \quad \rightarrow \int_0^\infty r\,exp\left(\frac{-r^2}{c_1^2x^2}\right)dr = \frac{c_1^2x^2}{2} \quad (10)$$

Therefore:

$$\dot{m}(x) = 2\pi\cdot\,7.75\,\rho(x)\frac{\sqrt{J/\rho(x)}}{x}\frac{c_1^2x^2}{2} = 2\pi\frac{7.75c_1^2}{2}\rho(x)\sqrt{J/\rho(x)}\ \cdot\ x \quad (11)$$

The last expression shows that for $\rho = constant,\ \dot{m}(x)$ is a linear function with a slope:

$$\alpha = 2\pi\,\frac{7.75c_1^2}{2}\rho\sqrt{J/\rho} = 2\pi\frac{7.75c_1^2}{2}\sqrt{\rho\cdot J} \quad (12)$$

The essential features of a circular jet is that the jet speed is proportional to $1/x$ while the mass rate of the entrained air is proportional to x.

For a vertical jet in the atmosphere the air density is not constant. Numerical calculations provide the total mass rate $\dot{m}(x)$ as a function of the height using the temperate lapse rate to calculate the density.

Assumptions and a Procedure for Numerical Calculations for a Circular Jet produced by a Turbofan Jet Engine

We choose to use the high bypass ratio Pratt & Whitney JT9D turbofan for our analysis. Specifications are the following:

- **Type:** High bypass two-spool turbofan engine
- **Diameter:** 92.3 in (fan tip 2.34 meter)
  Dry weight: 8,608 lb (3,905 kg)

Maximum thrust: 46,300 to 50,000 lbf (205.95 to 222.41 kN) take-off
Overall pressure ratio: overall 23.4:1 (Fan 1.64:1)
Bypass ratio: 5.0:1

The take-off thrust is 205,000 N. Since turbofan cannot operate for long periods in take-off conditions let's assume that we operate the turbofan to provide 50% of the takeoff thrust or $J = 100{,}000\ N$.

We need an estimate for the mass rate through both the fan and the core operating at sea level providing $100{,}000\ N$ thrust.

The total cross section area of both the fan and the core are:

$$A_t = \frac{\pi D^2}{4} = \frac{\pi\, 2.34^2}{4} = 4.30\ m\text{^}2$$

The bypass ratio is 5:1. Therefore, the fan cross section is 5/6 of total cross section area while the diffuser cross section is 1/6 of total area.

$$A_f = \frac{5}{6}\, 4.30 = 3.58\ \ m^2 \qquad A_c\ =\ \frac{1}{6}\, 4.30 = 0.716\ \ m^2$$

A_f And A_c are the cross sections of the fan and the core (diffuser).

Let's assume that since the nozzle jet speed is higher than the speed of the jet produced by the fan that the core provides a jet speed which is 80% higher than that of the fan.

This problem has been solved numerically to provide:

The mass rate through the fan is $517\frac{kg}{sec}$

The jet speed produced by the fan is $118\frac{m}{sec}$

The mass rate through the core is $186\frac{kg}{sec}$

The nozzle (core) jet speed is $212\frac{m}{sec}$

After the two jets are mixed and merged the combined jet momentum is equal to the sum of the two separate jets.

The combined jet speed is $143\frac{m}{sec}$

Total mass rate $705\frac{kg}{sec}$

Summary: The jet produced by our chosen Pratt & Whitney JT9D turbofan operating at 50% of the takeoff power produces Thrust of $\mathbf{100,000\ N}$, mass rate of $\mathbf{705\ {}^{kg}/_{sec}}$ and a jet speed of

$\mathbf{143\ {}^{m}/_{sec}}$.

Calculation Procedure

Step 1

For the sake of analysis the characteristics of our jet are the following:

- Assume the combustion gas is dry air.
- For the sake of example, I don't concern myself (now) with the turbomachinery that produces the jet flow, only with the jet flow itself. When I speak about energy or power, I mean the power of the jet flow and not the power of the machinery that produces this jet flow. An analysis and a design of specific a turbomachinery equipment will be done later on.

A suggested procedure for numerical calculations is provided below. I provide detailed calculations for the jet at heights 50 and 1,000 meters.

Step 2

Assign a vertical profile of the temperature lapse rate, and pressure. Calculate the density of the air as a function of the height x.

(Although our envisioned system may be used to address atmospheric inversions, it is impossible to know in advance the height, temperature gradient and the thickness of the atmospheric inversion. Therefore, we use standard temperature lapse rate atmospheric conditions. Later on, we plan to develop software that will incorporate atmospheric inversions in the numeric procedure based on on-time remote sensing of specific inversions).

Example A

Let's calculate the jet starting at $x = 25 \cdot d_0 \cong 50\ meter$. Substituting in eq. (7):

$$U_m(x = 50) = c_2\,\frac{\sqrt{{}^{J}/_{\rho(x)}}}{x} = 7.75\,\frac{\sqrt{{}^{100,000}/_{\rho(20)}}}{50} = 44.3\ \ m\,s^{-1} \qquad (13)$$

$\rho(50) \cong \rho(0) = 1.225\, kg\, m^{-3}$ (14)

(Density of air at standard atmospheric conditions. $T = 288K,\ \rho(0) \cong 1.225\ kg\, m^{-3}$).

The last result is significant: It shows that the jet speed at the centerline has been reduced from $143\, m\, s^{-1}$ into a Gaussian profile where the maximum speed in the centerline is $44.3\, m\, s^{-1}$ so the flow becomes incompressible quite immediately upon ejection from the jet engine. In this specific example we calculated that the air mass flowing through the JT9D turbofan is:

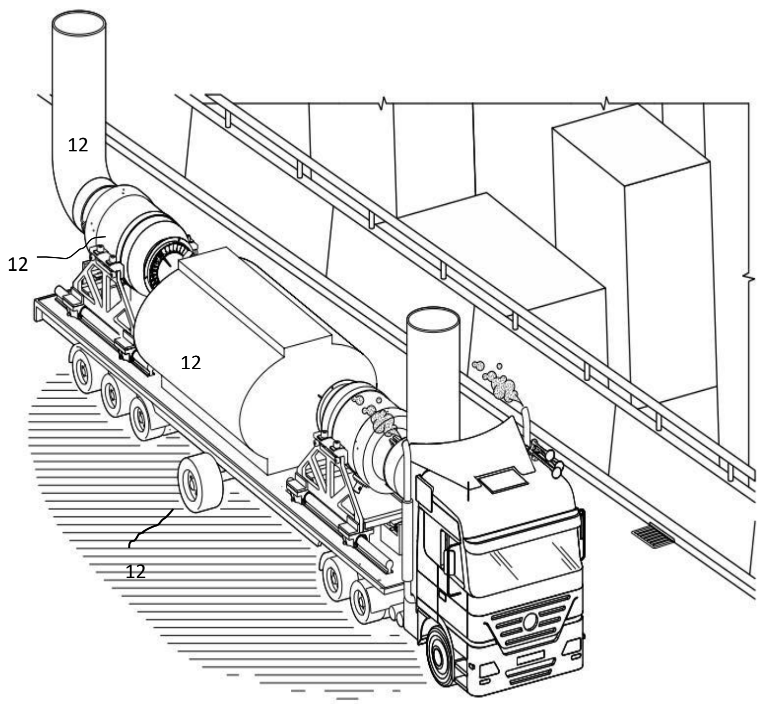

Figure 3: Shrounded turbofan engines for smog mitigation

$\dot{m}_{tr} \cong 705\, kg\, sec^{-1}$ (15)

Calculate the total mass flow rate of the free jet including the entrained air of the Gaussian profile at $x = 50\, meter$. Using eq. (5):

$$U(50,r) = U_m(50) \cdot exp\left[\frac{-r^2}{0.103^2 \cdot 50^2}\right] = 44.3 \cdot exp\left[\frac{-r^2}{26.52}\right] \quad (16)$$

Using (9), (10) and (11) the total mass flow rate at $x = 50\ meter$ is:

$$\dot{m}(x = 20) = 2\pi \cdot\ 44.3\rho(50) \int_0^\infty r \cdot exp\left[\frac{-r^2}{26.5}\right] dr \quad (17)$$

Using the integral: $$\int_0^\infty r \cdot exp\left[\frac{-r^2}{a}\right] dr = \frac{a}{2} \quad (18)$$

We obtain: $\dot{m}(x = 50) = 4{,}519\ kg\ sec^{-1} = 6.41\ \dot{m}_{tr}$ - The ratio of the total mass flow rate of the jet and its entrained air at $x = 50\ meter$ to the mass rate through the turbofan is 6.41.

Using (4) check the momentum flow of the free jet at $x = 50\ meter$:

$$J(x = 50) = 2\pi \cdot\ \rho(50) \cdot 44.3\text{^}2 \int_0^\infty r \cdot exp\left[\frac{-2r^2}{0.103^2 \cdot 50^2}\right] dr \quad (19)$$

$$\int_0^\infty r \cdot exp\left[\frac{-2r^2}{0.103^2 \cdot 50^2}\right] dr = \int_0^\infty r \cdot exp\left[\frac{-r^2}{13.26}\right] dr = 6.63$$

The Thrust therefore is:

$$J = 2\pi \cdot\ 1.225 \cdot 44.3^2 \cdot 6.63 = 100{,}000\ N$$

As expected the momentum flow at $X = 50\ meter$ of the free jet is the same as that produced by the turbofan engine.

Comment: Because the entrained ambient air in the free jet is 6.41 times higher than mass flow of through the turbofan, it was a reasonable assumption that the temperature and pressure of the Gaussian jet at $x = 50\ meter$ is that of the ambient air due to an intense turbulent mixing. This assumption seems to be correct for any distance from the nozzle where the velocity of the jet becomes Gaussian.

<u>Example B</u>

Calculate the speed of the jet and its total mass flow rate at $x = 1{,}000\ meter$.

The density is calculated using the temperature lapse rate for dry air:

$$\frac{\rho(1{,}000)}{\rho_0} = \left(\frac{T(1{,}000)}{T_0}\right)^{\frac{g}{R_{air}\mu} - 1} \qquad \mu = 0.0065\ K\ m^{-1} \quad (20)$$

$\frac{\rho(1{,}000)}{\rho_0} = 0.91 \quad \rightarrow \rho(1{,}000) = 1.115\ kg\ m^{-3}$

Substituting:

$$U_m = c_2 \frac{\sqrt{J/\rho(1{,}000)}}{x} = 7.75 \frac{\sqrt{100{,}000/1.115}}{1{,}000} = 2.32\ ms^{-1}$$

$$U(1{,}000, r) = 2.32 \cdot \exp\left[\frac{-r^2}{0.103\hat{}2 \cdot 1{,}000\hat{}2}\right]$$

The total mass flow rate at $x = 1{,}000\ meter$:

$$\dot{m}(x = 1{,}000) = 2\pi \cdot\ 2.32 \cdot \rho(1{,}000) \int_0^\infty r \cdot exp\left[\frac{-r^2}{10{,}609}\right] dr$$

$$\dot{m}(x = 1{,}000) = 86{,}172\ kg\ sec^{-1} \cong 122 \cdot \dot{m}_{tr}$$

Checking the momentum flow at a height $x = 1{,}000\ meter$:

$$J = 2\pi \cdot\ 2.32\hat{}2 \cdot \rho(1{,}000) \int_0^\infty r \cdot exp\left[\frac{-2r^2}{10{,}609}\right] dr \cong 100{,}000\ Newton$$

As expected the momentum flow is the same as that produced by the jet engine.

Jet Numerical Calculations

The procedure above has been used to calculate $U_m(x)$ and $\dot{m}(x)$ up to a height of $3{,}000\ meter$ see figure 4, 5, 6 and 7. Due to scaling, the graphs are divided into velocity and total mass rate in the range of $x = 0 - 500\ meter$ and a second set for $x = 500 - 3{,}000\ meter$.

Figures 5 and 6 show that the mass rate of entrained air/initial mass rate of turbofan vs. height is (almost) a linear function with a slope:

$$\alpha = \frac{\dot{m}_{total}/\dot{m}_{tr}}{x} \cong 0.12\ m^{-1} \qquad (21)$$

It means that each 1 meter of the jet column at any height captures entrained air in the amount 0.12 of the mass flown through the jet engine. The value of α here is estimated from the numerical calculations and is slightly different than the value of α given by equation (12) that is derived for a jet expanding into constant density fluid.

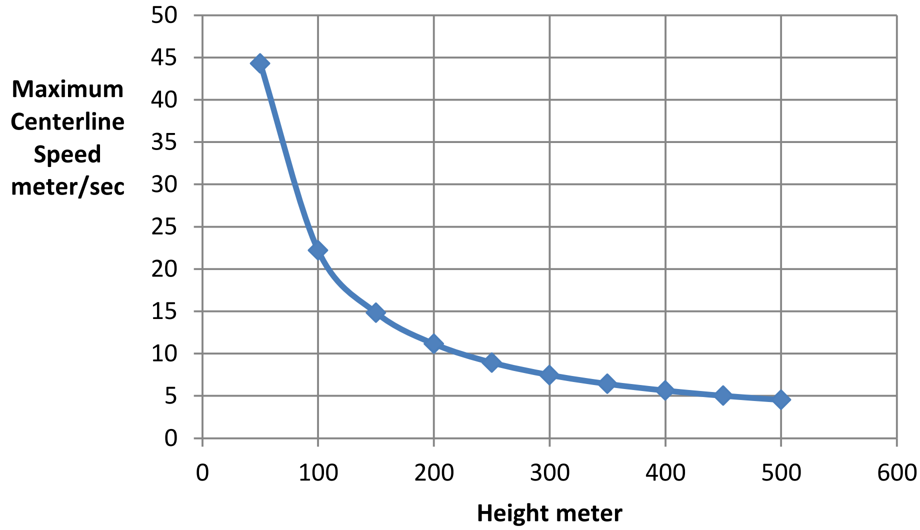

Figure 4: Jet Maximum Centerline Velocity Vs. Height; Initial Jet Velocity is $143\ m/sec$.

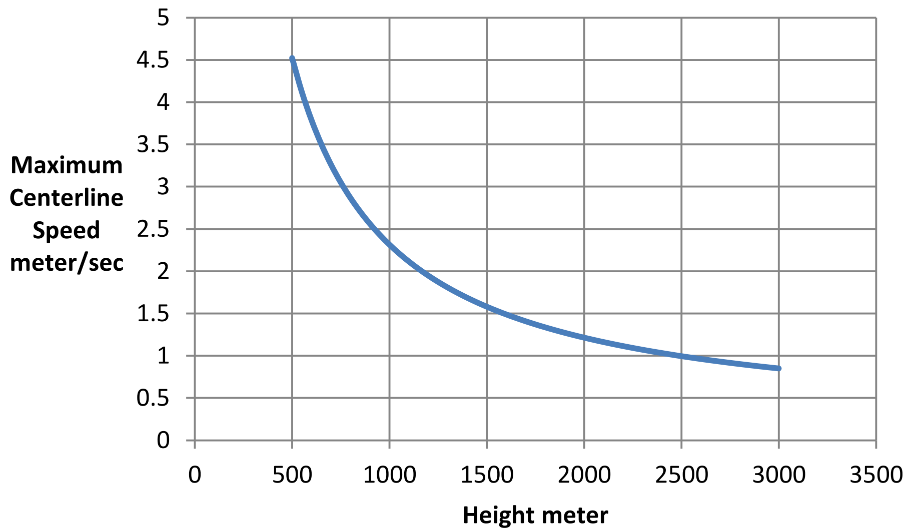

Figure 5: Jet Maximum Centerline Velocity Vs. Height; Initial Jet Velocity $143\ meter/sec$.

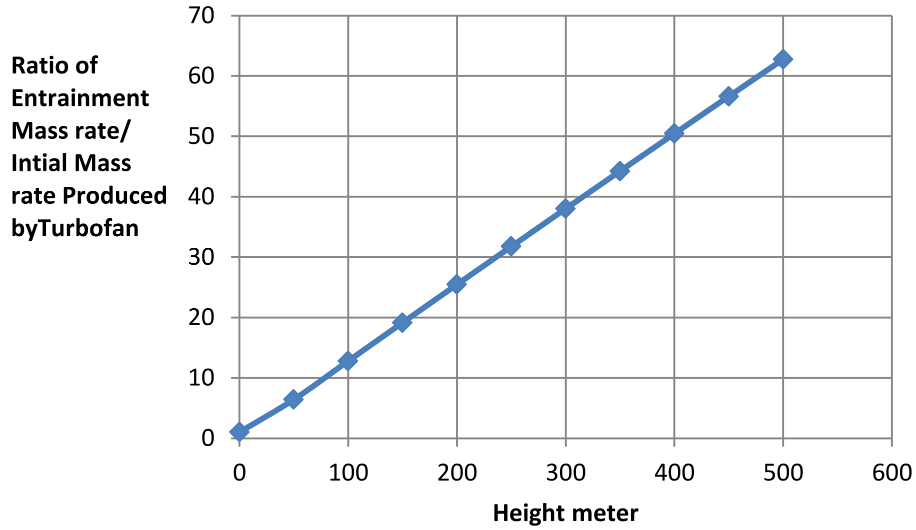

Figure 6: Jet mass flow rate due to entrainment. Turbofan initial mass flow rate is $705\ kg\ sec^{-1}$..

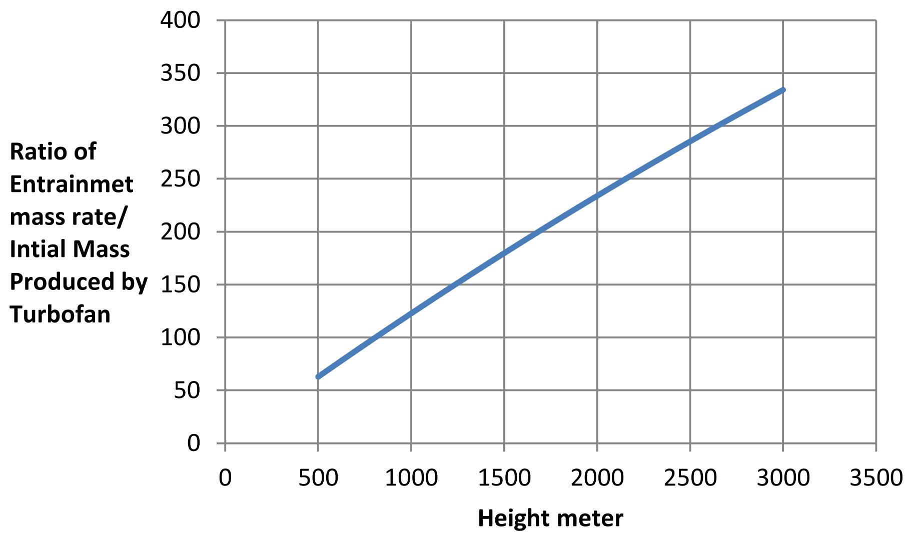

Figure 7: Jet mass flow rate due to entrainment. Engine initial mass flow rate is $705\ kg\ sec^{-1}$.

Propulsion Leverage

Figure 3 shows that at height of $50\ meter$ the maximum velocity of the jet Gaussian profile is

$44.3\ m\ s^{-1}$ while the jet speed produced by the turbofan engine it is $143 m\ s^{-1}$. It is clear that, although, the momentum flow is equal for both heights the jet loses immediately much of its power.

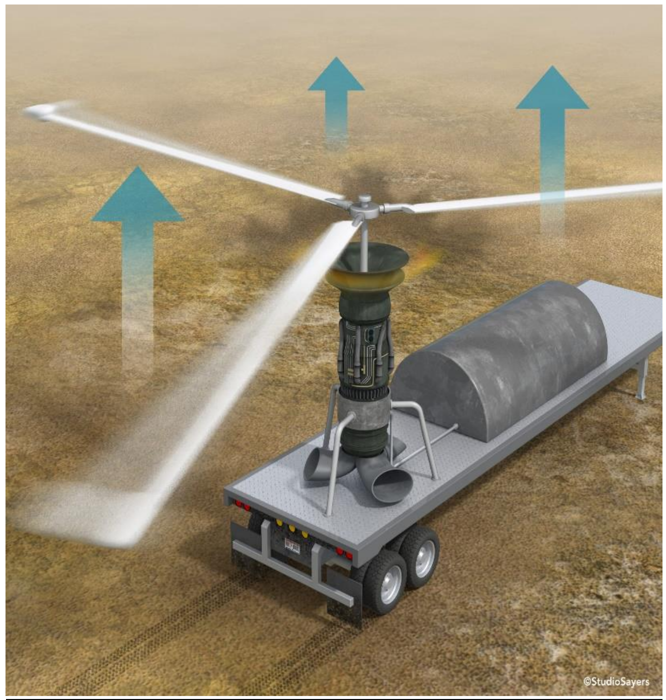

Figure 8: Conceptual conversion of turbojet into turboprop (or turboshaft) system

It is possible however to device a system starting at a velocity of $44.3\ m\ s^{-1}$ instead $143\ m\ s^{-1}$ providing the same momentum flow as in the high speed jet but the necessary power would be smaller than what the faster jet requires. The proposed theoretical concept is for the sake of analysis of a turboprop or a turboshaft as show in Figure 7. The following analysis and calculations are rudimentary, and precise analysis will require information on specific turbomachinery hardware and design.

The momentum flow (thrust) of a jet is:

$$J = \dot{m} \cdot V \qquad \text{[Newton]} \qquad (22)$$

Where $\dot{m}$the mass is rate of the jet and V is its velocity.

The power of a jet is:

$$P = \dot{m}\frac{V^2}{2} \qquad \text{[Watts]} \qquad (23)$$

Let's assume a fast jet produced by a turbojet where its mass rate is $\dot{m}_a$ and its velocity is V_a so its momentum flow is $J_a = \dot{m}_a \cdot V_a$ and its power is $P_a = \dot{m}_a \cdot \frac{V_a^2}{2}$.

Now assume a second jet where its mass rate is increased by a factor of k, $k > 1$ so its mass rate is $\dot{m}_b = k\dot{m}_a$. Assume also that this new jet velocity is reduced to $V_b = \frac{V_a}{k}$. Therefore the momentum flow (thrust) of the new jet is:

$$J_b = k\dot{m}_a \cdot \frac{V_a}{k} = \dot{m}_a V_a = J_a \tag{24}$$

The momentum flow of the new slower jet is the same as that of the faster jet produced by the turbojet. What is the power of the new slower velocity jet?

$$P_b = k\dot{m}_a \cdot \frac{\left(V_a/k\right)^2}{2} = k\dot{m}_a \frac{V_a^2}{2k^2} = \quad k\dot{m}_a \cdot \frac{V_a^2}{2}\frac{1}{k^2} \tag{25}$$

Therefore:

$$P_b = \frac{P_a}{k} \tag{26}$$

The necessary power for the slower jet is reduced by a factor k in comparison to the faster jet although both jets provide the same thrust. The insight gained here is that it is preferable to apply a jet with lower velocity but higher mass rate so the power required for the slower jet is reduced.

A second case is where two jets c and d have the same power but one jet velocity is lower than the second faster jet. How the performances of the slower jet are improved?

The power of the two jets is:

$$Pc = \dot{m}_c \frac{V_c^2}{2} = P_d = \dot{m}_d \frac{V_d^2}{2} \tag{27}$$

Assume that the velocity of the slower jet is: $V_d = \frac{V_c}{k}$ $\quad k > 1$

Substituting in (26):

$$\dot{m}_d \frac{V_d^2}{2} = \frac{\dot{m}_d}{2}\left(\frac{V_c^2}{k^2}\right) = \dot{m}_c \frac{V_c^2}{2} \tag{28}$$

Or:

$$\frac{\dot{m}_d}{\dot{m}_c} = k^2 \tag{29}$$

And the momentum flow (thrust) of the slower jet becomes:

$$J_d = \dot{m}_d V_d = k^2 \dot{m}_c \frac{V_c}{k} = k\, J_c \tag{30}$$

The momentum flow of the slower jet is higher by a factor k in comparison to the faster jet.

These principles are applied in a turbofan where large air mass rate is flown through the fan but the jet velocity produced by the fan is lower than the fast jet produced by a turbojet. In fact, a turboprop uses the same principles so the power required for the jet produced by a turboprop is smaller than what would be required for a jet produced by a turbojet that provides the same thrust. Even better than a turboprop is a turboshaft where the shaft powers a large rotor such as used in helicopters.

The best, therefore, is that we find retired propulsion systems of helicopters. This we hope to do for the pilot testing. If we reach operational commercial phases, however, we will need hundreds if not thousands of such systems. The problem is that there are not that many retired helicopters around. At that stage we might want to convert a retired turbojet or a turbofan into a turboshaft.

In fact, Pratt and Whitney did this exactly this when they converted the JT12 turbojet into Pratt & Whitney JFTD12 turboshaft engine. This turboshaft engine is used to power large helicopters such as the SkyCraine. When we reach commercial and operational phases we might commission Pratt and Whitney to help us to design a conversion of JT8D and JT9D, two of the most produced engines into a new turboshaft. The new design might be manufactured in China.

The conversion of a turbojet or turbofan into a turboshaft requires a gear to reduce the high RPM of the turbine. Aviation gears are complex and expensive since they should be lightweight and fit for aviation. Our system does not fly so we can use heavy and bulky gear that is used today in any gas turbine that power electric generators. There are many such retired gears around.

In the example of the turbofan above we assume that the lower velocity of the jet at $x = 50\,meter$ generated by a turboshaft with a large diameter rotor is $V_b = 44.3\ ms^{-1}$ while the velocity of the faster jet produced by the turbofan is $V_a = 143\ ms^{-1}$. The factor k is therefore:

$$k = \frac{143}{44.3} = 3.23 \qquad (31)$$

Using (26):

$$P_b = \frac{P_a}{k} = \frac{P_a}{3.23} = 0.31 P_a \qquad (32)$$

This would mean the power necessary for an ideal turboshaft obtained by the conversion of a turbofan jet is only 31% of the power required for the faster jet produced by the turbofan.

The conclusion is that the power required for the production of the slower jet (and for the same thrust) is decreased when the upwelling air by a turboprop or turboshaft that powers a large rotor, corresponding to a higher height on the constant momentum flow vertical free jet cone. Starting a jet with lower velocity reduces the power needed for the turbomachinery that produces the slower jet.

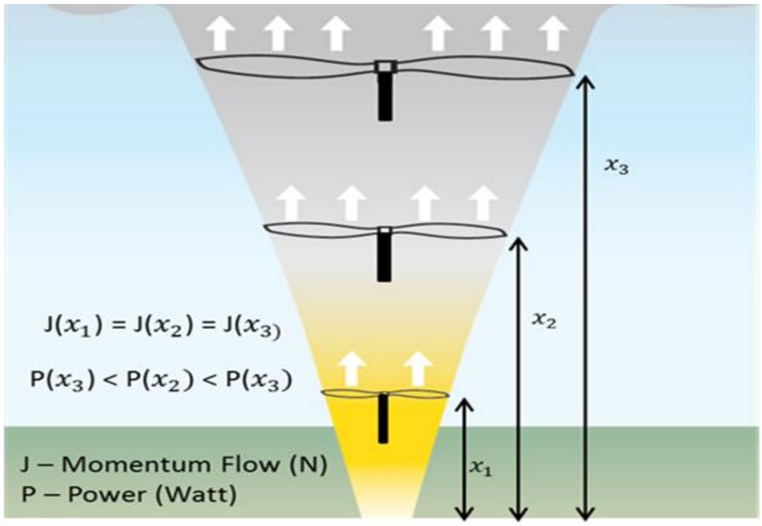

Figure 9: A schematics for large diameter rotors replacing the turbojet along the cone of a fast jet produced by a turbojet. Each rotor provides the same thrust as the turbojet. The rotor could provide the same momentum flow but requires less power. .

This analysis is rudimentary and is an approximate since we did not take in account the Gaussian nature of the jet flow velocity but only its centerline velocity.

There are two insights gained here:

A turboprop (or turboshaft) enables us to reduce the initial velocity of the jet reducing the power needed, while maintaining the momentum flow (thrust);

The velocity of a jet produced from a turbofan engine and from a turboprop used in aviation must be greater than the flight speed of the airplane (otherwise thrust is not produced). Because our turboprop system is stationary, we are able to further reduce the velocity of the turboprop jet, maintaining its thrust but reducing the power requirements.

The conceptual turboprop system shown in Figure 7 and its analysis has a few structural limitations. But the principles discussed here can be implemented in a variety of simpler and inexpensive embodiments.

Planar Free Jet

A planar jet has a rectangular cross section and an aspect ratio of a least 15:1. In a planar jet (as in a circular jet) the momentum flow is preserved. However, the entrainment of air into the jet is slower than in a circular jet, where the entrained air flows radially toward the jet. The result is that the jet speed for a planar jet is reduced with height slower than in a circular jet.

The equations governing the planar jet are:

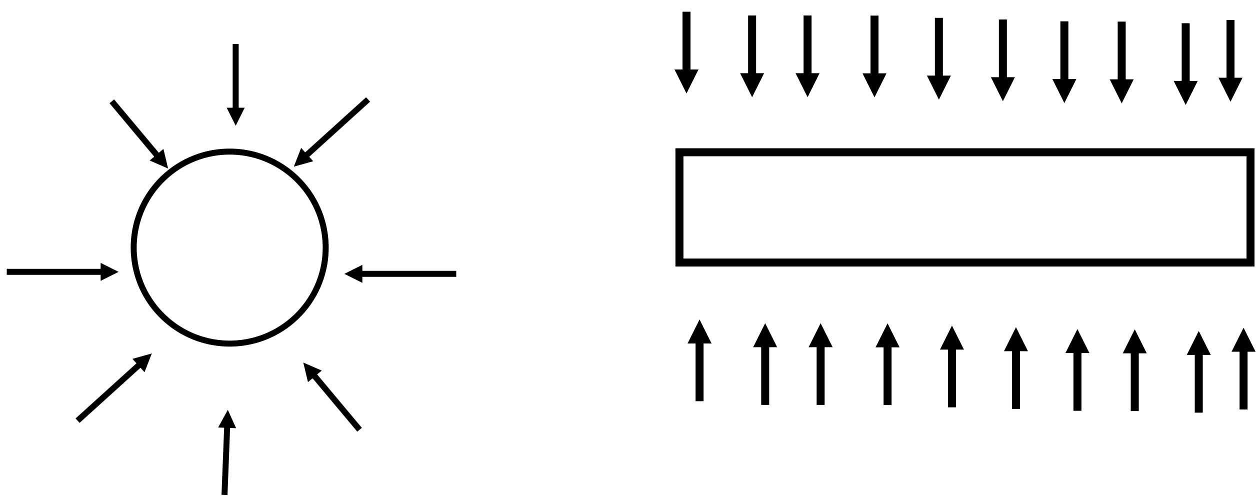

Figure 10: Entrainment into circular jet versus planar jet.

$$U_{mp} = d_2 \frac{\sqrt{J'/\rho(x)}}{x^{0.5}} \qquad (33)$$

In equation (32) J' is the momentum flow per unit length in $\lfloor Nm^{-1} \rfloor$ and d_2 is constant. The sub notation p is for “planar”.

The velocity profile for a planar jet is:

$$U(x,y) = U_{mp} \cdot exp\left[\frac{-y^2}{d_1^2\, x^2}\right] \qquad (34)$$

Where d_1 is a constant. d_1 and d_2 were found in the same manner as for circular jet to be (Ref. 1, 3):

$$d_1 = 0.132 \qquad d_2 = 2.46 \qquad (35)$$

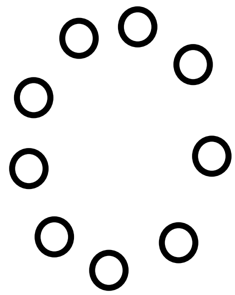

Figure 11: Circular cluster of N jet engines

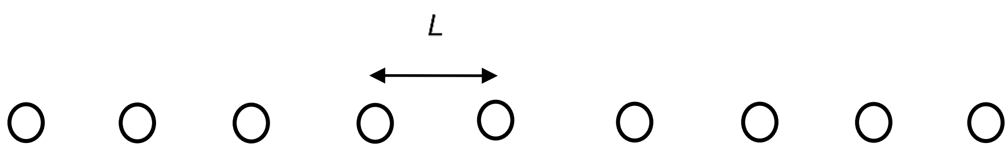

Figure 12: Planar configuration of N jet engines

For a planar jet the divergence of the jet is:

$$y_{Ump0.5} = 0.11 \cdot x \qquad (36)$$

Arrangement of Multiple Jet Engines

In some cases, the application of many jet engines at one site may be necessary. The physical arrangement of the engines has three potential configurations:

1. The jet engines are far away from each other and each jet does not influence each other. This configuration is difficult logistically, since each engine will require its platform.

2. Use the engines as a circular cluster on one platform. In this case, the cluster can be viewed as one large engine that provides momentum flow of N engines.

In this case:

$$U_{mN} = c_2 \frac{\sqrt{{NJ_{single}}/{\rho(x)}}}{x} = c_2 \sqrt{N} \frac{\sqrt{{J_{single}}/{\rho(x)}}}{x} \quad (37)$$

It is possible to show that the air mass rate is also multiplied by $\sqrt{N}$ in comparison to the mass flow rate of a single engine.

3. Have the N engines arranged in a straight row. In this case, the arrangement can be viewed as a planar jet where the momentum flow per unit depth is equal to the momentum flow of each engine divided by the distance L between the engines or:

$$J' = \frac{J_{single}}{L} \quad (38)$$

Substituting (36) into (32):

$$U_{mp}(x) = d_2 \frac{\sqrt{{J_{single}}/{L\rho(x)}}}{x^{0.5}} \quad (39)$$

The essential difference between a circular and a planar jet arrangements is the dependency of the jet maximum velocity on the height. The speed is proportional to ${1}/{x^{0.5}}$ while for a circular arrangement it is proportional to ${1}/{x}$. In the planar case, although the velocity of the jet may be higher (for certain L) the total mass flow may be lower since it is proportional to $x^{0.5}$.

These calculations could provide optimization for the best arrangement at various times during operation. If multiple jets engines are used, it might be better to use one configuration at one time while at another time another configuration.

References:

1. Abramovich, G.N., *The Theory of Turbulent Jets*, The MIT Press, 1963.

2. Rajaratnam, N., Turbulent Jets, Elsevier Scientific Publishing, 1976.

3. Rodi, W., in *Studies in Convection*, P. 79-165; Launder, B.E., Editor, Academic Press, London, 1975.