A Preliminary Assessment of Inducing Anthropogenic Tropical Cyclones Using Compressible Free Jets and the Potential for Hurricane Mitigation

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Abstract. We have conceptually studied the potential for mitigation of natural hurricanes by inducing anthropogenic perturbations prior to or in front of an advancing hurricane. We propose actual hardware for the task. It consists of multiple jet engines mounted on barges or ships that will be dispatched to strategic locations in the ocean where the sea surface temperature is high and the vertical temperature profile and atmospheric conditions are such that the potential for development of a hurricane or tropical storm is high. The engines will direct compressible high momentum, high-speed free jets skyward causing entrainment of even larger amounts of additional air to form plumes and updrafts. The unstable humid updraft will itself produce conditions for additional entrainment and evolution of tropical cyclones. These anthropogenic perturbations will extract enthalpy from the ocean, cooling the ocean surface and depriving the advancing natural hurricane of its needed thermal energy.

The barrage of hurricanes and adverse impacts on human life and property in the Southeastern United States and Caribbean Basin during 2004-2005 has revived an interest in hurricane mitigation (Hoffman 2004, 2002). Reputable atmospheric scientists have developed a few ideas for hurricane mitigation over the past 60 years (Gray et al. 1976; Simpson 1981). Only two programs have involved actual field or laboratory experiments (Willoughby et al. 1985; Alamaro 2001). The most well known was the Stormfury project that lasted for more than 20 years, under which NOAA used cloud seeding by silver iodide to try to nucleate supercooled water in the hurricane's clouds. The hypothesis was that the heat of fusion released upon nucleation would increase the hurricane eyewall diameter, leading to a decrease in the maximum wind speed. Through radar observations it was eventually discovered that the clouds contain ice and little or no supercooled water, so the project was abandoned (Willoughby et al. 1985). Another attempt was undertaken at the Air-Sea Interaction Laboratory at the Massachusetts Institute of Technol-ogy, where a monolayer film was used to retard evaporation in a wind wave tank in which the airflow over the water surface was comparable to that of hur-ricane (Alamaro 2001). The hypothesis was that spreading a monolayer film on the ocean in the front of the hurricane would retard the evaporation that fuels the hurricane with latent heat. Unfortunately, at a wind speed of about 10 m/sec or higher the film tends to break apart and becomes immersed in the water due to high-speed airflow and wave action, and loses its effectiveness (See: http://alamaro.home.comcast.net/Evaporationretardation.htm).

We propose to induce atmospheric perturbations in front of or prior to an advancing hurricane or potentially dangerous cyclone. These induced perturbations will extract enthalpy from the ocean surface, leading to a decrease in the sea surface temperature (SST). As such, the approaching naturally occurring hurricane will be deprived of its source of enthalpy. It is hypothesized that the hurricane intensity will then be much reduced prior to landfall.

Compressible free jets generated by multiple jet engines mounted on barges or ships will induce the perturbations. They will be dispatched to strategic locations in the ocean during an advancing hurricane. Alternatively, the jet generator vessels will continuously patrol the western Tropical Atlantic, inducing cyclones during the hurricane season to reduce the SST up to a few hundred miles from the shoreline.

The proposed method is analogous to backfires created by firefighters when confronting an advancing firewall. Small and controlled fires are started in front of the advancing, uncontrolled and larger fire. By the time the main fire advances, its fuel supply has been consumed causing it to be reduced in intensity and if properly executed, extinguished. Just as firefighters maintain distance between the backfires and the larger fire so they do not merge, it would be necessary to keep a distance between the induced cyclones and the natural hurricane so they do not merge to form a larger hurricane.

Compressible Free Jets and Plumes. A free jet is an unbounded flow of one fluid into another and is generated by pressure difference at the orifice of the jet. A plume is an updraft of air due to buoyancy forces (Lee and Chu 2003). Free jets are usually turbulent and turbulent mixing causes transport of momentum, energy and species to the surrounding fluid. The effective mass transfer rate of the jet is increased with the distance from the jet orifice due to entrainment as its velocity is reduced. The momentum flux of a free jet is preserved during entrainment.

The airspeed in the updraft of a hurricane is on the order of 1-10 m/sec. We have concluded that generating such incompressible low airspeed by a free jet is impractical since to achieve a substantial air mass flow and momentum a fan of impractical large cross sectional area would be required. Moreover, the momentum flux imparted to a jet is proportional to the square of its speed and as a result the mass rate of the entrained air is proportional to the square of the initial jet speed.

Therefore, we suggest the use of a compressible high-speed free jet generated by surplused, retired or decommissioned jet engines. For example, a Pratt & Whitney JT3D-3B jet engine that was used to power airplanes such as the Boeing 707, or its military equivalent the TF33-3 engine that was used to power the B-52 provide roughly 50,000 N of thrust, with an air mass flow of approximately 110 Kg/sec and a compressible free jet speed is about 460 m/sec. Twenty such jet engines mounted on a barge will provide a total momentum flux of about 10^6 N. By the time the entrained jet airspeed decreases to 10 m/sec and subsequently to 1.0 m/sec the ascending entrained air mass will have reached a flow rate of 10^5 Kg/sec and 10^6 Kg/sec, respectively. This ascend-ing air mass rate is due to the initial momentum pro-vided by the jet engines and does not include plumes. It is roughly 3-4 orders of magnitude smaller than the ascending air mass in the updraft of fully developed hurricane[1]. It is expected that if this is done at loca-tions where the SST is higher than 26^0C - 27^0C, where the atmosphere is humid and the gradients along height of potential temperature is negative causing instability, the free jet will eventually grow to produce large and unstable plumes and updrafts of tropical storm or hurricane proportions.

The approximated chemical reaction of combustion in the jet engine is:

$$CH_2 + 3/2\ O_2 = H_2O + CO_2$$

The combustion gas excluding nitrogen (which is not affected by the combustion) and unburned oxygen is a mixture of water vapor and carbon dioxide, the molecular weight of which is 31, slightly heavier than that of air. Since the ratio of combustion gas to air in the jet of turbofan jet engine is small, it is not expected that the increase in the molecular weight of the jet will impede its upward motion. Turbulent mixing with ambient air would result in an immediate reduction of the jet density approaching that of the ambient air. Furthermore, injection of water in the high temperature jet will add water vapor to the jet and increase the humidity of the entrained air, increasing the buoyancy.

Implementation. We conceptually studied two scenarios for implementation of the proposed technology. In the first, the natural hurricane's path is predicted. Barges towed by ships as shown in Figure 1 carry the jet engines and fuel, and are dispatched in the Western Tropical Atlantic or the Caribbean, westward of the advancing hurricane. Because there is always some uncertainty about the track of a hurricane that is traveling westward, multiple free jet facilities will be situated in rows that are generally extended from south to north to account for any deviation from the predicted path of the hurricane.

The power and total energy of a tropical storm is less than the power and total energy of a hurricane. The power of moving air is proportional to the cube of its velocity. If the air speed of a tropical storm is (¾) that of a hurricane, the power per unit area of the ocean surface of a hurricane is approximately $1/(¾)^3$ or four times that of a tropical storm. In this example it will be required that approximately four tropical storms travel in front of a hurricane, and deprive it of enthalpy intake. Therefore, multiple tropical storms are required in advance of the advancing hurricane.

[1] An order of magnitude estimate for the mass rate of updraft air in a hurricane has been calculated based on the following approximations: Total power of a hurricane 10^{12} -10^{13} Watt. This estimate enables to find the rate of water vapor ascending using the latent heat of evaporation approximated as 2.5 10^6 J/kg. From knowing the rate of water vapor ascending and the water vapor density over SST at 27 Degree Cel-sius and assuming 100% relative humidity, we can arrive with an estimate for the rate of ascending air.

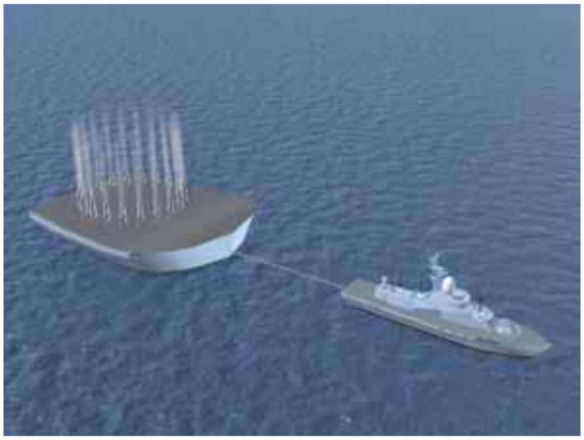

Fig. 1: Artistic view of multiple jet engines directing jet skyward for inducing anthropogenic atmospheric perturbations

Logistically, this scenario may be difficult to execute since once a hurricane is evolved there is not much time to determine where the barges should be dispatched, and for actual dispatching of the barges and free jet implementation. Even if successful, it may be possible that multiple tropical storms would reach the shoreline causing high winds and flooding. This, combined with the inherent uncertainties of the outcome of weather modification and the lack of confidence that indeed the induced cyclones will alter the trailing hurricane, may lead to an erosion of public support for the hurricane mitigation program.

In an alternative scenario, the SST of the ocean up to a distance of a few hundred miles from the shoreline will be kept lower by a few degrees during the hurricane season. For that, barges or ships equipped with compressible free jet systems will continuously patrol the oceans and be dispatched to locations where the SST is high. The initiation of tropical storms in this case will be done in advance of the evolution of hurricanes. The timing of the generation of anthropogenic tropical storms may include, in addition to hurricane mitigation, the requirements for rainfall at specific regions on land and the potential landfall locations of the hurricane. As more experience is gained, the proper spatial and temporal distribution of the anthropogenic storms can be developed to optimize effects and to avoid landfall of multiple storms at one location during a short period of time.

The preferred mode of operation in this case is the following: The first anthropogenic tropical storms will be initiated as close as possible to the shoreline regardless of an existing or advancing natural hurricane. By the time these induced man-made cyclones arrive on shore their intensity will be minimal and it is possible that they will decay before arriving on shore. At the same time or afterward, a second row of barges will start a second row of tropical cyclones Eastward of the initial row. The second row of cyclones will travel westward and eventually will travel over lower SSTs that had been caused earlier by the first row. At the same time or afterwards a third row of tropical cyclones will be started eastward of the second row, and so forth. It may be sufficient to cool the ocean surface by 2-3 degrees Celsius, up to a distance of a few hundred miles from the shoreline, to ensure that an advancing natural hurricane that is formed in the Mid or Eastern Atlantic would travel over lower SSTs, substantially reducing its potentially destructive energy before landfall.

The cost of full scale implementation to protect the Southern US, Caribbean Basin and Central America is estimated at $0.5 –0.75 billion per year. A substantial portion of this budget could be in-kind or overhead contributions by the military and various government agencies.

Atmospheric Conditions. Other or alternative effects and processes resulting from the anthropogenic cyclones may also contribute to the weakening of a natural hurricane as well. For example, the large-scale overturning subsidence associated with the secondary circulation of the anthropogenic storms may suppress convection and increase the vertical shear of horizontal wind in the inner core region of the approaching natural hurricane (Wang and Holland 1995). It is hypothesized that the hurricane intensity may also be reduced by this mechanism prior to landfall. Another possible consequence is the direct interaction between the anthropogenic perturbations and the natural hurricane. Such an interaction could change the track of the hurricane. It is possible that the anthropogenic cyclones could be "designed" to steer the natural hurricane from landfall at highly populated coastal regions or be steered back out to sea and away from any landfall (Hoffman 2004; Y. Wang, 2005 personal communication).

A more general application of anthropogenic modification is to modify the tropical atmosphere where it is most sensitive to small perturbations that will grow in amplitude and scale over several days to the point that some characteristics of a subsequent tropical cyclone will be altered. This does not necessarily involve direct creation of another tropical cyclone. It may instead involve tropical waves or other large-scale features. It may not be possible to entirely prevent a given tropical cyclone, but it might be possible to, for instance, alter the track of the storm slightly so as to miss large population centers or areas prone to devastating storm surge effects or inland flooding (C. Davis, 2005 personal communication). Calculations of sensitivity in the tropics are still in their undeveloped stages and require much better models than exist now, including calculations that are capable of resolving individual cumulus towers (or at least a much improved representation of their effects).

Many calculations of the response of the tropical atmosphere to small perturbations are required to understand what may result from a perturbation of the type envisioned to be created from arrays of jet engines. This study has many practical implications for understanding the sources of errors in dynamic prediction in the tropics, and so carries importance beyond anthropogenic modification scenarios (C. Davis, 2005 personal communication).

In the tropic oceans under direct solar radiation (the sun is not partly or completely covered by clouds) the stratification of the atmospheric layer over the sea surface is rather stable. Figure 2 shows average gradients of potential temperature $\Theta = T(1000/P)^{0.286}$ during 24 hours time, where T is the air temperature in ^{0}K, and P is the atmospheric pressure at the lev-el of measurements. Curve 1 in Figure 2 is for tropi-cal ocean areas. These data were obtained during numerous measurements made on expeditions in the Pacific Ocean and South China Sea (Pudov and Korolev 1990). Positive values for potential temperature gradient indicate stability while negative values indicate instability, a factor that may be necessary for inducing unstable updrafts by the proposed jet. The jet operation, therefore, may better be done at nights and/or in the presence of initial cloudiness of a fraction no less than 6-7.

The higher the relative humidity (f %), the lower the condensation height level where evaporation heat will be released from the ascending humid air. The height of the condensation level (h) can be roughly estimated as h = 22 (100-f %), where h is in meters above surface. For example, for a relative humidity of 80% the condensation level is about 400 meters. To reduce the condensation height it may be necessary to inject water into the exhaust nozzles of the jet engines.

The jet engines might be sufficient for disturbing the stability of the air layer near the water surface in the tropics, especially in the late afternoon or at night. In the extra-tropical zones it may be possible to create convective clouds with the help of vertical jets most of the day.

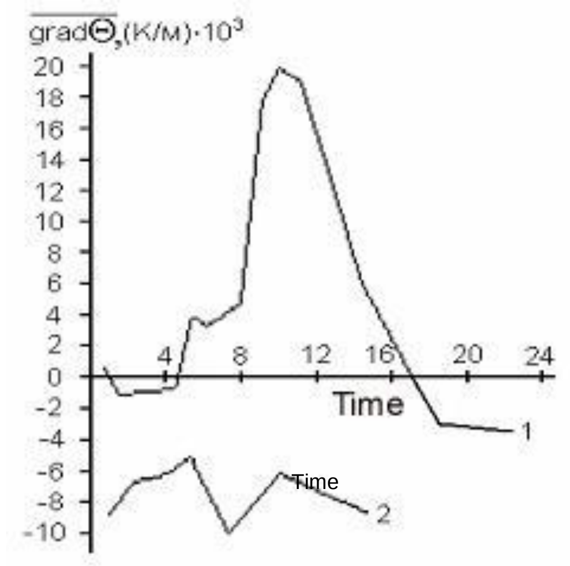

Fig. 2: Gradients of potential temperature along height vs. time; Curve 1 for over tropical ocean; Curve 2 for extra-tropical ocean (Pudov and Korolev 1990).

Legal and Policy Considerations. A report of The National Research Council states: “The Committee concludes that there is still no convincing scientific proof of the efficacy of intentional weather modification efforts.” (p. 3). The report also states: “If simple precipitating cloud systems cannot be modified in significant ways, it is very difficult to believe that a strongly organized large dynamic system such as a hurricane can be modified” (NRC 2003).

The fundamental problem of weather modification is that controlled experiments are difficult if not impossible. There is always uncertainty that the outcome of the modification, such as enhancing rainfall, is due to the modification or the natural variability of weather. This uncertainty is less of a problem in this proposal since if a tropical storm is generated consistently a few times after applying jet engines, cause and effect will become clear. Even if a well-supported theory of hurricane modification existed, the legal ramifications of weather modification on this scale are daunting. A few of the many possibilities include (R. Anthes, 2005 personal communication):

1. The storm is not modified at all, but some people perceive that it is, suffer personal damage or injury and file lawsuits.

2. The storm is modified according to theory, but still does significant damage and some people blame the modifiers on the damage,

even though the modification actually reduced overall damage and impact.

3. The modified storm produced “winners” and “losers” and the perceived “losers” sue. For example, what if the hurricane abruptly changed course? The people affected by the new course might well blame the modification effort and sue (R. Anthes, 2005 personal communication).

It is clear that first it would be necessary to further the theory and then to design experiments that do not have the potential to cause harm. Only then it would make sense to develop policy for international treaties to enable implementation under the supervision of international advisory committee, to assure public acceptance of hurricane modification. For example, according to future international treaties, hurricane damage will be compensated regardless if the hurricane has been modified or not. But suing will not be an option.

Pilot Development. Tropical cyclones involve complex fluid dynamic processes, including rotating and stratified flows, boundary layers, air-sea interaction and multiphase thermodynamics (Emmanuel 1991). It is impossible to scale down these processes in a laboratory experiment. The only avenue for development is to test the concept over the high seas. The first milestone would be the creation of an anthropogenic perturbation or tropical storm by a free compressible jet. This may be done anywhere in the ocean outside of the hurricane season, nominally before June or after November, to assure that the induced storm does not become a hurricane, or is perceived to have done so. To assure success of the pilot program we recommend employing as many jet engines as possible in the first trial runs and then to reduce the number if possible.

The projected cost of such a pilot project is estimated to be $25 - $40 million, most of which could be in-kind contributions from government agencies and the military. Old and retired jet engines can be donated by the Air Force for example, at scrap value or less. Airlines may also wish to donate such retired engines in exchange for write-offs against taxable income. Such arrangements can be made with the US Government and others, such as the Russian Government. The cost of a flight-worthy reliable jet engine is substantial but the reliability of the jet engines necessary for this project is not an issue. It would be entirely acceptable if 10 - 30% of the stationary jet engines used for the pilot program and subsequent implementation break down during operation.

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Appendix: Review of Free Jet and Strategies for the Application of Jet Engines

Introduction

The following is an outline for a new system for updrafts formation that could be used for the disruption of atmospheric inversions and a variety of applications, including smog and fog dispersion and weather modification. This analysis could lead to a few design alternatives.

The new concept will require turbomachinery that could consist of activated and retrofitted inexpensive retired de-commissioned jet engines. I will assume that all the turbomachinery steps are isentropic with efficiency of 100%.

Definition

The following contains basic information, assumptions and calculations for a free jet produced by a turbofan jet engine for a variety of applications.

A “Free Jet” is a flow of one fluid into another. The other is surrounding fluid at rest or in a motion, relative to the jet. Walls or ducts do not confine the free jet. The jet flow is impeded only by shear stress with the surrounding fluid.

The term “compressible” refer to the velocity of the free jet. For example, the jet can be made up of air, which is a compressible gas. However, the flow can be compressible or incompressible. In any flow where the Mach Number (M) is less than 0.3 the flow is incompressible. At $M > 0.3$ the flow is compressible. For air at ambient temperature, the flow is incompressible when the air speed is lower than approximately $100\ ms^{-1}$. The flow from a turbojet engines used in aircrafts is usually at around sonic speed, so it is initially compressible. Turbulent mixing and entrainment of air that joins the jet reduces the velocity of the compressible jet generated by a jet engine. The free jet, then, becomes incompressible quite immediately after being ejected from the nozzle.

The following analysis is intended to provide reasonable assumptions for a vertical free jet using a Pratt and Whitney JT9D turbofan. The engine can be placed horizontally and a duct could direct the jet upward. The methodology provided herein will be applied in the future to any specific propulsion systems and any specific design.

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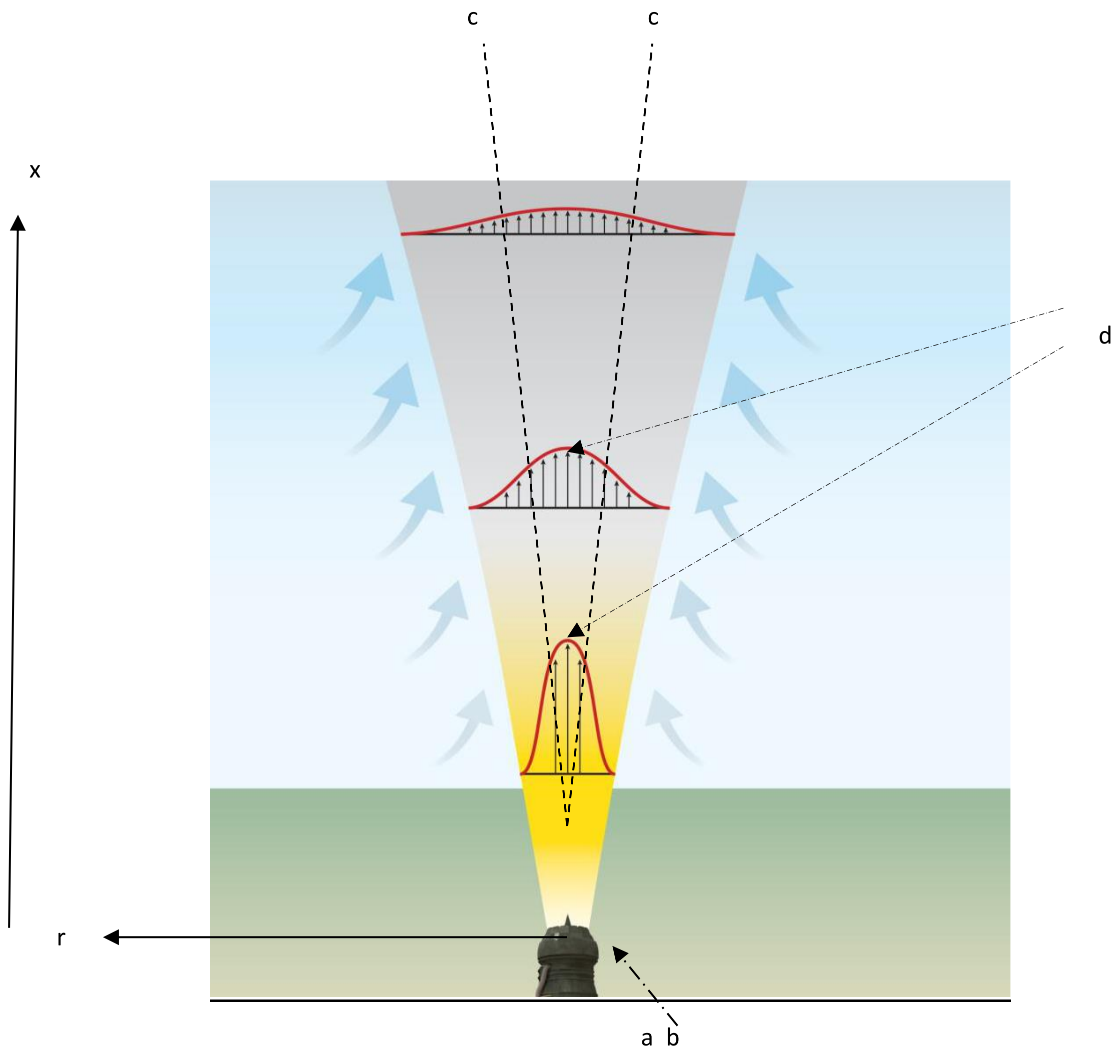

Figure 1: Geometry of a circular jet

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Geometry of Circular Jet

The schematic in Figure 2 describes the following:

a. Jet engine and a nozzle of diameter d_0 and cross sectional area A.

b. Approximately uniform velocity profile or "plug flow" at the jet nozzle exit.

c. Divergence lines where the flow velocity is half of the maximum velocity in the Gaussian profile or the lines of $U_{m\,0.5}$.

d. The maximum velocity U_m at the center of the circular Gaussian velocity profile.

For the vertical circular jet, the cylindrical coordinates are x and r where x is the height from the center of the nozzle. Assuming that the jet engine operates at a sea level, x is also the elevation above sea level.

Fundamental Assumptions

Buoyancy jets that extend to high altitudes, such as a few kilometers, have been studied extensively. Examples are the plums from the smokestack of a power plant. A horizontal convection jet produced by a jet engine has also been studied for understanding, for example, the turbulent wakes created by airplanes' jet engines during takeoff and landing. Vertical convection jets that extend to a height of a few kilometers have not been studied extensively. All or most past investigations pertain to jets that penetrate a constant density of fluid. In our case, the density of air is dependent on the height or distance from the jet origin.

Extensive analysis of a turbulent jet and its geometry is given in references 1, 2, and 3. For a circular jet, the radius at the points where the air velocity in the Gaussian profile is half of the maximum velocity is given by:

$$r_{U_{m\,0.5}} \cong 0.086 \cdot x \qquad (1)$$

The thrust produced by the jet engine is the momentum flow (also called momentum flux by a few authors) produced by the jet (ignoring the thrust that is generated by pressure imbalances). The momentum flow at the exit from the nozzle is:

$$J = \dot{m}U_n = \rho_n \, A_n \, U_n^2 \tag{2}$$

Where U_n is the speed at the nozzle exit from the engine which is approximated as uniform, A_n is the cross sectional area of the nozzle and ρ_n is the density of the gas at the nozzle exit. Here we ignore the portion of the thrust due to the fact that the static pressure of the flow is not necessarily equal to the ambient pressure.

The jet ejected from the nozzle forms a Gaussian velocity profile at certain distance from the nozzle which is estimated as $\approx (6-10)\, d_0$ while d_0 is the nozzle diameter. The jet ejected from an aviation jet engine nozzle could be supersonic, subsonic or sonic. The static pressure and the temperate of the gas ejected from the nozzle could be greater, lower or equal to the pressure of the ambient. All of these considerations are necessary for an aviation jet engine. However, our systems are stationary. At distance of $x \approx 10 \cdot d_0$ from the nozzle and by the time the flow becomes Gaussian, we assume (and show later) that the pressure and the temperature of the flow and its entrained air become equal to that of the ambient air due to entrainment and turbulent mixing.

Abramovich (ref. 1) has shown that for a circular axisymmetric jet:

$$\frac{\partial}{\partial x} J(x) = 2\pi\rho \frac{\partial}{\partial x} \int_0^{\infty} rU^2\,(x,r)dr = 0 \tag{3}$$

This and a control volume analysis shows that the momentum flow of the jet remains constant at any distance x (or height) from the engine nozzle.

The thrust generated by a jet engine at cruising airplane flight is comprised of the positive momentum flow of the combustion gas emitted from the engine's nozzle, minus the negative momentum flow (ram drug) of the air mass that enters the engine at the speed of the flight. In our case, however where the engine is stationary, the only momentum flow (equal to the thrust) is generated by the flow emitted from the nozzle. In this respect, the thrust generated by our jet is similar to the thrust during takeoff, which is much larger than the thrust of jet engine in a cruising flight.

Using a control volume analysis on the jet, it is possible to show that at any arbitrary distance x

from the jet origin the momentum flow of the jet is preserved. Therefore:

$$J = 2\pi\rho(x)\int_0^\infty rU^2\,(x,r)dr = constant \qquad (4)$$

for any value of x . In fact, this is the central feature of a free jet.

In Eq. (3) and (4) J is the momentum flow. In our turbofan used for analysis we assume that the density at the nozzle is equal to the density of dry air at standard sea level, where $T \cong 288K$ so $\rho_n = \rho_0 \cong 1{,}225\ kg/m\text{^}3$. $\rho(x)$ is the density at height x . $U(x,r)$ is the gas velocity which has a circular Gaussian velocity profile.

Description of a Circular Jet

The flow exiting from the jet engine nozzle has almost uniform velocity profile and therefore it is called "plug flow". Non-dimensional analysis provides the equations that approximate the subsequent Gaussian profile flow as:

$$U_m(x) = c_2\ \frac{\sqrt{J/_{\rho(x)}}}{x} \qquad (5)$$

Where $U_m(x)$ is the jet maximum velocity at the centerline, J is the momentum flow defined in Eq. (4), $\rho(x)$ is the air density as a function of height x and c_2 is some constant.

The velocity distribution as a function of r has several forms in different references. One convenient Gaussian velocity profile is:

$$U(x,r) = U_m(x) \cdot exp\left(\frac{-r^2}{a^2(x)}\right) \qquad For\ a\ circular\ jet \qquad a(x) = c_1 \cdot x \qquad (6)$$

c_1 is some constant.

To find c_1 and c_2 integrate the momentum flow expression:

$$J = 2\pi\rho(x)\int_0^\infty rU^2\,(x,r)dr = 2\pi\rho(x)\int_0^\infty r \cdot \left[c_2\ \frac{\sqrt{J/_{\rho(x)}}}{x} \cdot\ exp\left(\frac{-r^2}{a^2(x)}\right)\right]^2 \cdot dr$$

$$J = 2\pi\rho(x)c_2^2\ \frac{J}{\rho(x)x^2}\int_0^\infty r \cdot exp\left(\frac{-2r^2}{c_1^2x^2}\right)dr = 2\pi\rho(x)c_2^2\ \frac{J}{\rho(x)x^2} \cdot \frac{c_1^2x^2}{4}\ \rightarrow$$

$$\rightarrow \quad c_1^2 c_2^2 = \frac{2}{\pi} \rightarrow c_2 = \frac{1}{c_1}\sqrt{2/\pi} \tag{7}$$

c_1 has been found empirically to be 0.103 so $c_2 = 7.75$ (Ref. 3).

Therefor for a circular Jet:

$$U(x,r) = 7.75\,\frac{\sqrt{J/\rho(x)}}{x} \cdot exp\left(\frac{-r^2}{a^2(x)}\right) = 7.75\,\frac{\sqrt{J/\rho(x)}}{x} \cdot exp\left(\frac{-r^2}{0.103^2 x^2}\right) \tag{8}$$

The total mass rate of the jet as a function of x is:

$$\dot{m}(x) = 2\pi\rho(x)\int_0^\infty r\,U(x,r)dr = 2\pi \cdot\ 7.75\,\rho(x)\frac{\sqrt{J/\rho(x)}}{x}\int_0^\infty r \cdot\ exp\left(\frac{-r^2}{c_1^2 x^2}\right)dr \tag{9}$$

Using:

$$\int_0^\infty r \cdot exp\left[\frac{-r^2}{a}\right]dr = \frac{a}{2} \qquad \rightarrow \int_0^\infty r\,exp\left(\frac{-r^2}{c_1^2 x^2}\right)dr = \frac{c_1^2 x^2}{2} \tag{10}$$

Therefore:

$$\dot{m}(x) = 2\pi \cdot\ 7.75\,\rho(x)\frac{\sqrt{J/\rho(x)}}{x}\frac{c_1^2 x^2}{2} = 2\pi\frac{7.75 c_1^2}{2}\rho(x)\sqrt{J/\rho(x)} \cdot x \tag{11}$$

The last expression shows that for $\rho = constant,\ \dot{m}(x)$ is a linear function with a slope:

$$\alpha = 2\pi\,\frac{7.75 c_1^2}{2}\rho\sqrt{J/\rho} = 2\pi\frac{7.75 c_1^2}{2}\sqrt{\rho \cdot J} \tag{12}$$

The essential features of a circular jet is that the jet speed is proportional to $1/x$ while the mass rate of the entrained air is proportional to x.

For a vertical jet in the atmosphere the air density is not constant. Numerical calculations provide the total mass rate $\dot{m}(x)$ as a function of the height using the temperate lapse rate to calculate the density.

Assumptions and a Procedure for Numerical Calculations for a Circular Jet produced by a Turbofan Jet Engine

We choose to use the high bypass ratio Pratt & Whitney JT9D turbofan for our analysis. Specifications are the following:

- **Type:** High bypass two-spool turbofan engine
- **Diameter:** 92.3 in (fan tip 2.34 meter)
  Dry weight: 8,608 lb (3,905 kg)
  Maximum thrust: 46,300 to 50,000 lbf (205.95 to 222.41 kN) take-off
  Overall pressure ratio: overall 23.4:1 (Fan 1.64:1)
  Bypass ratio: 5.0:1

The take-off thrust is 205,000 N. Since turbofan cannot operate for long periods in take-off conditions let's assume that we operate the turbofan to provide 50% of the takeoff thrust or $J = 100{,}000\ N$.

We need an estimate for the mass rate through both the fan and the core operating at sea level providing $100{,}000\ N$ thrust.

The total cross section area of both the fan and the core are:

$$A_t = \frac{\pi D^2}{4} = \frac{\pi\, 2.34^2}{4} = 4.30\ m\text{^}2$$

The bypass ratio is 5:1. Therefore, the fan cross section is 5/6 of total cross section area while the diffuser cross section is 1/6 of total area.

$$A_f = \frac{5}{6}\,4.30 = 3.58\ \ m^2 \qquad A_c\ =\ \frac{1}{6}\,4.30 = 0.716\ \ m^2$$

A_f And A_c are the cross sections of the fan and the core (diffuser).

Let's assume that since the nozzle jet speed is higher than the speed of the jet produced by the fan that the core provides a jet speed which is 80% higher than that of the fan.

This problem has been solved numerically to provide:

The mass rate through the fan is $517\frac{kg}{sec}$

The jet speed produced by the fan is $118\frac{m}{sec}$

The mass rate through the core is $186\frac{kg}{sec}$

The nozzle (core) jet speed is $212\,\frac{m}{sec}$

After the two jets are mixed and merged the combined jet momentum is equal to the sum of the two separate jets.

The combined jet speed is $143\,\frac{m}{sec}$

Total mass rate $705\,\frac{kg}{sec}$

Summary: The jet produced by our chosen Pratt & Whitney JT9D turbofan operating at 50% of the takeoff power produces Thrust of $\boldsymbol{100,000\ N}$, mass rate of $\boldsymbol{705\ {}^{kg}/_{sec}}$ and a jet speed of

$\boldsymbol{143\,{}^{m}/_{sec}}$.

Calculation Procedure

Step 1

For the sake of analysis the characteristics of our jet are the following:

- Assume the combustion gas is dry air.
- For the sake of example, I don't concern myself (now) with the turbomachinery that produces the jet flow, only with the jet flow itself. When I speak about energy or power, I mean the power of the jet flow and not the power of the machinery that produces this jet flow. An analysis and a design of specific a turbomachinery equipment will be done later on.

A suggested procedure for numerical calculations is provided below. I provide detailed calculations for the jet at heights 50 and 1,000 meters.

Step 2

Assign a vertical profile of the temperature lapse rate, and pressure. Calculate the density of the air as a function of the height x.

(Although our envisioned system may be used to address atmospheric inversions, it is impossible to know in advance the height, temperature gradient and the thickness of the atmospheric inversion. Therefore, we use standard temperature lapse rate atmospheric conditions. Later on, we plan to develop software that will incorporate atmospheric inversions in the numeric procedure based on on-time remote sensing of specific inversions).

Example A

Let's calculate the jet starting at $x = 25 \cdot d_0 \cong 50\, meter$. Substituting in eq. (7):

$$U_m(x = 50) = c_2 \frac{\sqrt{J/\rho(x)}}{x} = 7.75 \frac{\sqrt{100{,}000/\rho(20)}}{50} = 44.3 \;\; m\, s^{-1} \tag{13}$$

$$\rho(50) \cong \rho(0) = 1.225\, kg\, m^{-3} \tag{14}$$

(Density of air at standard atmospheric conditions. $T = 288K,\ \rho(0) \;\cong\; 1.225 \;\; kg\, m^{-3}$).

The last result is significant: It shows that the jet speed at the centerline has been reduced from $143\, m\, s^{-1}$ into a Gaussian profile where the maximum speed in the centerline is $44.3\, m\, s^{-1}$ so the flow becomes incompressible quite immediately upon ejection from the jet engine. In this specific example we calculated that the air mass flowing through the JT9D turbofan is:

$$\dot{m}_{tr} \;\cong 705\, kg\, sec^{-1} \tag{15}$$

Calculate the total mass flow rate of the free jet including the entrained air of the Gaussian profile at $x = 50\, meter$. Using eq. (5):

$$U(50, r) = U_m(50) \cdot exp\left[\frac{-r^2}{0.103^2 \cdot 50^2}\right] \; = 44.3 \cdot exp\left[\frac{-r^2}{26.52}\right] \tag{16}$$

Using (9), (10) and (11) the total mass flow rate at $x = 50\, meter$ is:

$$\dot{m}(x = 20) = 2\pi \cdot\; 44.3\rho(50) \int_0^\infty r \cdot exp\left[\frac{-r^2}{26.5}\right] dr \tag{17}$$

Using the integral:

$$\int_0^\infty r \cdot exp\left[\frac{-r^2}{a}\right] dr = \frac{a}{2} \tag{18}$$

We obtain: $\dot{m}(x = 50) = 4{,}519\, kg\, sec^{-1} = 6.41\, \dot{m}_{tr}$ - The ratio of the total mass flow rate of the jet and its entrained air at $x = 50\, meter$ to the mass rate through the turbofan is 6.41.

Using (4) check the momentum flow of the free jet at $x = 50\, meter$:

$$J(x=50) = 2\pi \cdot \ \rho(50) \cdot 44.3\text{^}2 \int_0^\infty r \cdot exp\left[\frac{-2r^2}{0.103^2 \cdot 50^2}\right] dr \tag{19}$$

$$\int_0^\infty r \cdot exp\left[\frac{-2r^2}{0.103^2 \cdot 50^2}\right] dr = \int_0^\infty r \cdot exp\left[\frac{-r^2}{13.26}\right] dr = 6.63$$

The Thrust therefore is:

$$J = 2\pi \cdot \ 1.225 \cdot 44.3^2 \cdot 6.63 = 100{,}000\ N$$

As expected the momentum flow at $X = 50\ meter$ of the free jet is the same as that produced by the turbofan engine.

Comment: Because the entrained ambient air in the free jet is 6.41 times higher than mass flow of through the turbofan, it was a reasonable assumption that the temperature and pressure of the Gaussian jet at $x = 50\ meter$ is that of the ambient air due to an intense turbulent mixing. This assumption seems to be correct for any distance from the nozzle where the velocity of the jet becomes Gaussian.

<u>Example B</u>

Calculate the speed of the jet and its total mass flow rate at $x = 1{,}000\ meter$.

The density is calculated using the temperature lapse rate for dry air:

$$\frac{\rho(1{,}000)}{\rho_0} = \left(\frac{T(1{,}000)}{T_0}\right)^{\frac{g}{R_{air}\mu}-1} \qquad \mu = 0.0065\ K\ m^{-1} \tag{20}$$

$$\frac{\rho(1{,}000)}{\rho_0} = 0.91 \quad \rightarrow \rho(1{,}000) = 1.115\ kg\ m^{-3}$$

Substituting:

$$U_m = c_2 \frac{\sqrt{J/\rho(1{,}000)}}{x} = 7.75 \frac{\sqrt{100{,}000/1.115}}{1{,}000} = 2.32\ ms^{-1}$$

$$U(1{,}000, r) = 2.32 \cdot \exp\left[\frac{-r^2}{0.103\text{^}2 \cdot 1{,}000\text{^}2}\right]$$

The total mass flow rate at $x = 1{,}000\ meter$:

$$\dot{m}(x = 1{,}000) = 2\pi \cdot\ 2.32 \cdot \rho(1{,}000) \int_0^{\infty} r \cdot exp\left[\frac{-r^2}{10{,}609}\right] dr$$

$$\dot{m}(x = 1{,}000) = 86{,}172\ kg\ sec^{-1} \cong 122 \cdot \dot{m}_{tr}$$

Checking the momentum flow at a height $x = 1{,}000\ meter$:

$$J = 2\pi \cdot\ 2.32\text{^}2 \cdot \rho(1{,}000) \int_0^{\infty} r \cdot exp\left[\frac{-2r^2}{10{,}609}\right] dr \cong 100{,}000\ Newton$$

As expected the momentum flow is the same as that produced by the jet engine.

Jet Numerical Calculations

The procedure above has been used to calculate $U_m(x)$ and $\dot{m}(x)$ up to a height of $3{,}000\ meter$ see figure 4, 5, 6 and 7. Due to scaling, the graphs are divided into velocity and total mass rate in the range of $x = 0 - 500\ meter$ and a second set for $x = 500 - 3{,}000\ meter$.

Figures 5 and 6 show that the mass rate of entrained air/initial mass rate of turbofan vs. height is (almost) a linear function with a slope:

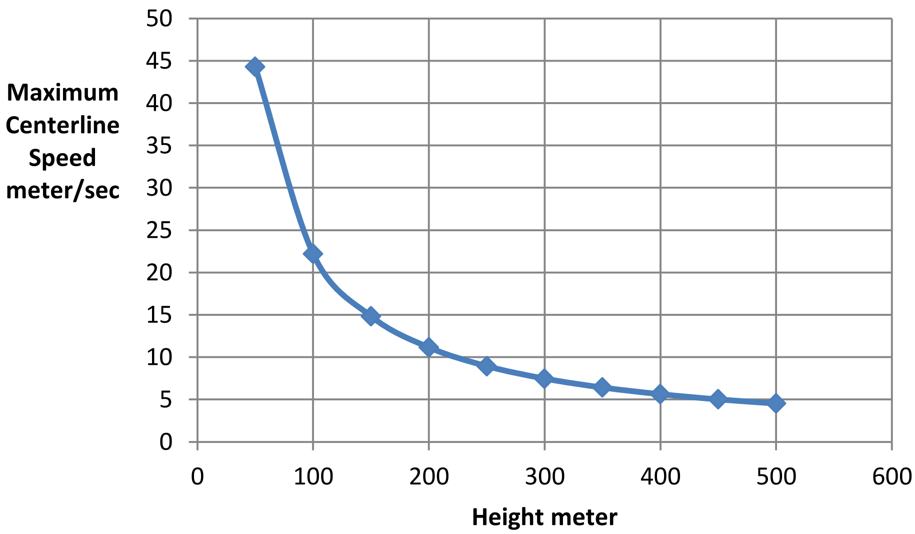

Figure 2: Jet Maximum Centerline Velocity Vs. Height; Initial Jet Velocity is $143\ m/sec$.

$$\alpha = \frac{\dot{m}_{total}/\dot{m}_{tr}}{x} \cong 0.12\ m^{-1} \tag{21}$$

It means that each 1 meter of the jet column at any height captures entrained air in the amount 0.12 of the mass flown through the jet engine. The value of α here is estimated from the numerical calculations and is slightly different than the value of α given by equation (12) that is derived for a jet expanding into constant density fluid.

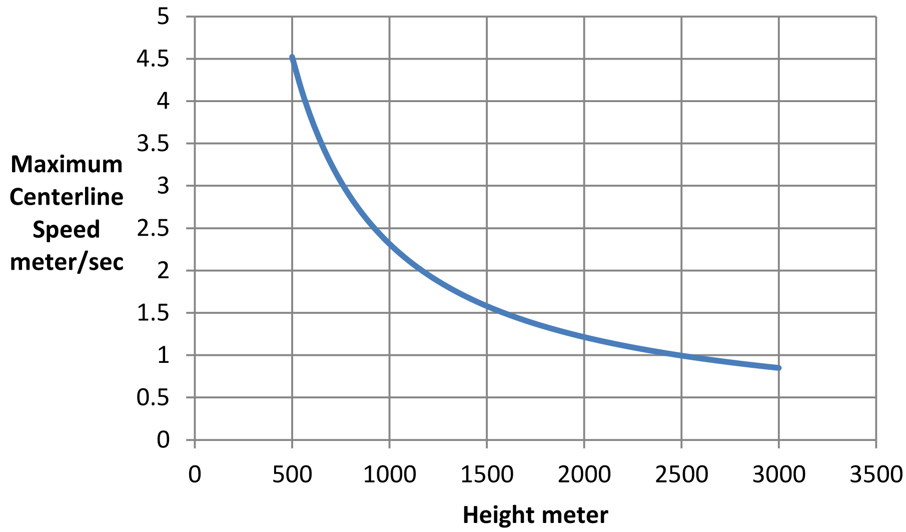

Figure 3: Jet Maximum Centerline Velocity Vs. Height; Initial Jet Velocity $143\ meter/sec$.

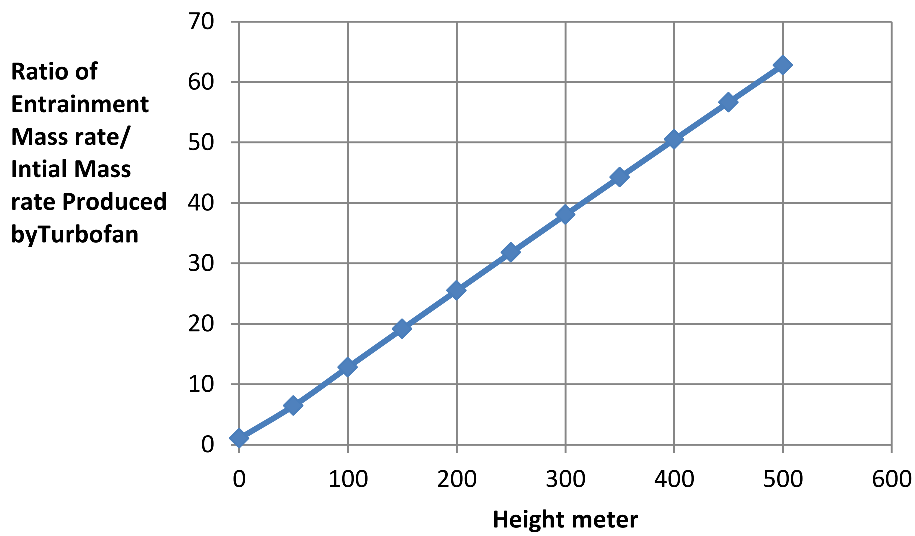

Figure 4: Jet mass flow rate due to entrainment. Turbofan initial mass flow rate is $705\ kg\ sec^{-1}$..

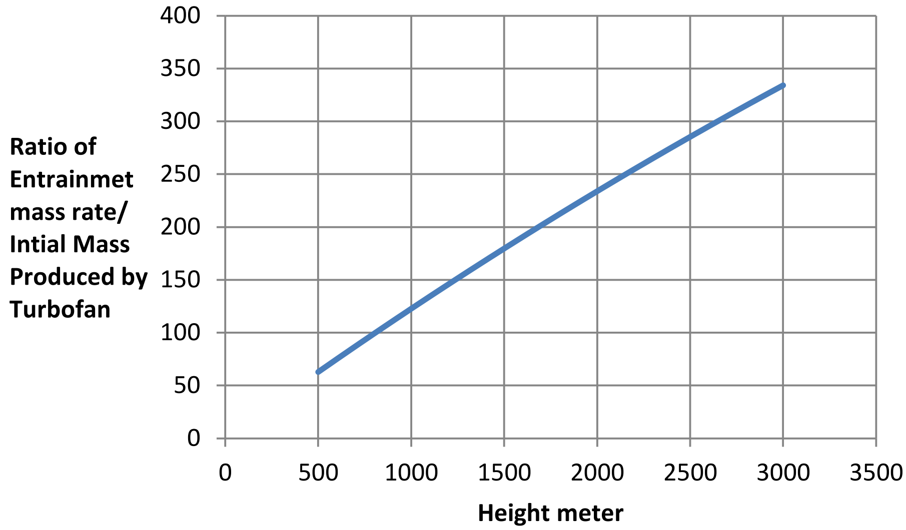

Figure 5: Jet mass flow rate due to entrainment. Engine initial mass flow rate is $705\ kg\ sec^{-1}$.

Planar Free Jet

A planar jet has a rectangular cross section and an aspect ratio of a least 15:1. In a planar jet (as in a circular jet) the momentum flow is preserved. However, the entrainment of air into the jet is slower than in a circular jet, where the entrained air flows radially toward the jet. The result is that the jet speed for a planar jet is reduced with height slower than in a circular jet.

The equations governing the planar jet are:

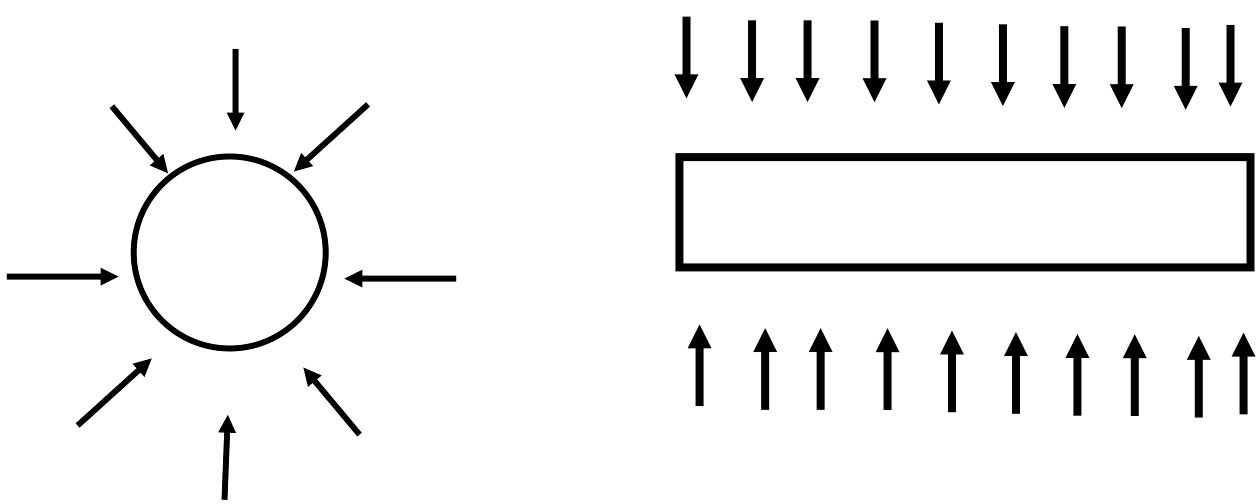

Figure 10: Entrainment into circular jet versus planar jet.

$$U_{mp} = d_2 \frac{\sqrt{J'/\rho(x)}}{x^{0.5}} \tag{33}$$

In equation (32) J' is the momentum flow per unit length in $[Nm^{-1}]$ and d_2 is constant. The sub notation p is for "planar".

The velocity profile for a planar jet is:

$$U(x,y) = U_{mp} \cdot exp\left[\frac{-y^2}{d_1^2\, x^2}\right] \tag{34}$$

Where d_1 is a constant. d_1 and d_2 were found in the same manner as for circular jet to be (Ref. 1, 3):

$$d_1 = 0.132 \qquad d_2 = 2.46 \tag{35}$$

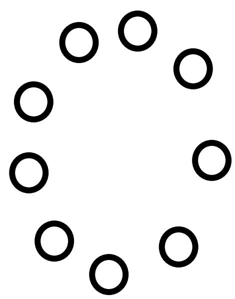

Figure 11: Circular cluster of N jet engines

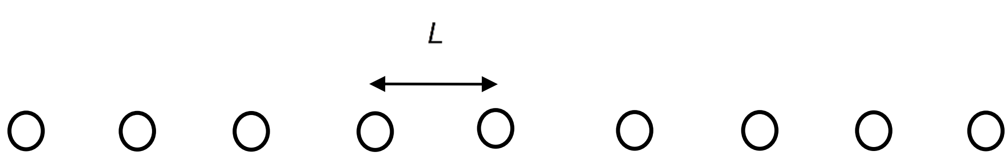

Figure 12: Planar configuration of N jet engines

For a planar jet the divergence of the jet is:

$$y_{Ump0.5} = 0.11 \cdot x \qquad (36)$$

Arrangement of Multiple Jet Engines

In some cases, the application of many jet engines at one site may be necessary. The physical arrangement of the engines has three potential configurations:

1. The jet engines are far away from each other and each jet does not influence each other. This configuration is difficult logistically, since each engine will require its platform.

2. Use the engines as a circular cluster on one platform. In this case, the cluster can be viewed as one large engine that provides momentum flow of N engines.

In this case:

$$U_{mN} = c_2 \frac{\sqrt{{NJ_{single}}/{\rho(x)}}}{x} = c_2 \sqrt{N} \frac{\sqrt{{J_{single}}/{\rho(x)}}}{x} \qquad (37)$$

It is possible to show that the air mass rate is also multiplied by $\sqrt{N}$ in comparison to the mass flow rate of a single engine.

3. Have the N engines arranged in a straight row. In this case, the arrangement can be viewed as a planar jet where the momentum flow per unit depth is equal to the momentum flow of each engine divided by the distance L between the engines or:

$$J' = \frac{J_{single}}{L} \qquad (38)$$

Substituting (36) into (32):

$$U_{mp}(x) = d_2 \frac{\sqrt{{J_{single}}/{L\rho(x)}}}{x^{0.5}} \qquad (39)$$

The essential difference between a circular and a planar jet arrangements is the dependency of the jet maximum velocity on the height. The speed is proportional to ${1}/{x^{0.5}}$ while for a circular arrangement it is proportional to ${1}/{x}$. In the planar case, although the velocity of the jet may be higher (for certain L) the total mass flow may be lower since it is proportional to $x^{0.5}$.

These calculations could provide optimization for the best arrangement at various times during operation. If multiple jets engines are used, it might be better to use one configuration at one time while at another time another configuration.

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3. Rodi, W., in *Studies in Convection*, P. 79-165; Launder, B.E., Editor, Academic Press, London, 1975.